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Petri Nets And Their Application to Command And Control Systems

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DEPARTMENT OF DEFENCE
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ABSTRACT

Technical Report

Petri nets are directed graphs which were designed to model discrete event systems with concurrence and resource sharing. This makes them a useful method of graphically representing command and control systems. In this document Petri nets and some of their extensions are explained in detail. Coloured Petri nets, an extension of Petri nets, with timed transitions are used to model a command and control system.

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PETRI NETS AND THEIR APPLICATION TO COMMAND AND CONTROL SYSTEMS

Executive Summary

Systems Simulation and Assessment group is currently developing software to be used in the modelling of command and control systems. This software will be used for analysis of existing and possible command and control systems.

This document introduces the concept of using Petri nets as a tool to model command and control systems. The document first introduces Petri nets, concentrating on those aspects that are relevant for the modelling and analysis of command and control systems. This is followed by a study of a particular command and control system which illustrates the applicability of Petri nets as a tool for this type of modelling.

One of the main problems with using Petri nets is the models become complex when used to represent large systems. This has been overcome by extending the Petri net representation to allow more complex elements to be represented in a simpler form. This extension and the ease with which concurrence, synchronisation, and resource sharing can be represented by Petri nets makes them an ideal method of representing command and control systems.

Systems Simulation and Assessment group plan to apply Petri nets two ways. In the representation of nodes in an interactive simulation and as a method of analysing complete command and control systems. As part of these aims research into analysis techniques is being conducted as well as the development of Petri net simulation tools.

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Fred Bowden completed his Bachelor of Science at Murdoch University in 1989 majoring in Mathematics and Physics. He joined Combat Systems Division (now part of Information Technology Division) in 1990. In 1993 Fred completed a First Class Honours degree in Applied Mathematics at the University of Adelaide. He is currently studying for a doctorate relating to the application of extended Petri nets to military Command, Control, Communications and Intelligence systems.

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ABBREVIATIONS

C2	Command and Control
CPN	Coloured Petri Net
CTMC	Continuous Time Markov Chain
PN	Petri Net
SAM	Surface to Air Missile

1 INTRODUCTION

The aim of this document is to introduce the idea of using Petri nets (PN) in the modelling and analysing of command and control (C2) systems. The work reported here was done under task ADF 93/237, the command, control, communications and intelligence simulation task, and also contributed to an honours thesis in applied mathematics.

The modelling of C2 systems is a rapidly expanding field, mainly motivated by the U.S. Department of Defence over the last ten to fifteen years. In this time, many different methods of modelling C2 have been developed:

- Time line models, [10] and [22].
- Dynamic models, in which various methods of dynamic analysis, both classical and modern, are applied. Methods such as thermodynamics, [21], discrete state Markov processes, [39-41], statistical mechanics, [15-17], chaos theory, [9], [11], and [46-47], and adaptive control, [42].
- Conflict and combat models such as Lanchester models, stochastic combat models, and game theory. These models have been used in the past to model combat and can be further adapted to include C2 aspects, [43].
- PN, which model the data flow through the C2 system.

Before a modelling method is chosen the modeller must establish what type of model is most applicable for the system being modelled. The U.S. Joint Chiefs of Staff in [1] define C2 as:

> "The exercise of authority and direction by a properly designated commander over assigned forces in the accomplishment of his mission. Command and control functions are performed through an arrangement of personnel, equipment, communications, facilities, and procedures which are employed by a commander in planning, directing, coordinating and controlling forces and operations in the accomplishment of his mission."

This definition illustrates that C2 systems can be thought of as complex event systems which involve concurrent and parallel activities, synchronisation of events and resource sharing. This makes PN ideally suited for modelling such systems as I will demonstrate.

The PN method involves the definition of each of the main components of the system and modelling the information flow through these elements, to the required level of detail for the study being performed. For example, this may mean modelling a decision maker as a single process, by its exact definition, or by a generic means such as that developed by Levis in [24].

This document comprises two parts. Section 2 introduces the theory of Petri nets (PN), and sections 3 to 5 show how PN can be applied to a C2 system.

2 PETRI NETS

PN were originally developed by Carl Petri in his doctorial thesis in 1962 [35] for use in the modelling of computer systems. Since their development PN have been used to model many different systems including computer circuits, as in [34] and [36], assembly lines, see [28], flexible manufacturing systems, [44] and more recently C2, [5], [6], [20], [23-26], [31], [37], and [45]. In this section the basic ideas behind PN will be explained. Some of the properties of PN are outlined, other properties may be found in references [14], [27], [31], [33-34], [36], and [38]. The use of reachability trees, the PN matrix representation, and the conversion of PN into Continuous Time Markov Chains (CTMC) are explained as methods for analysing PN. It should be noted there are other methods of analysis, such as those presented in [12] and [38]. Finally, some extensions to PN are introduced. These include coloured tokens and some variations to arcs.

2.1 Ordinary Petri nets

A PN is a directed graph with two types of nodes: *places* and *transitions.* Pictorially, places are indicated by circles and represent entities such as conditions and buffers. Transitions are displayed on the graph as bars and represent concepts in the real system such as processors, algorithms, and events. The nodes are joined by one of two types of directed arcs: *input arcs* and *output arcs.* An input arc goes from a place to a transition. The set of places with input arcs going to a given transition are called the transition's *input places.* An output arc runs from a transition to a place. The set of places with output arcs coming from a particular transition are called the transition's *output places.* It should be noted that arcs can only go from a place to a transition or visa versa. *Tokens* make up the final element in a PN. Tokens are represented graphically by identical dots and can only be found in places. The movement of tokens between places is controlled by the transitions of the PN. In a model, the position of the tokens defines the state of the system, defining situations, such as availability of resources, satisfied conditions and items in a buffer. Each place is mapped to the number of tokens in it by a function, defined as the *marking.* A transition is said to be *enabled* if and only if all of its input places contains at least one token for each input arc going from the place to the transition. When an enabled transition is activated, changing marking, it is said to *fire.* Upon firing, the transition removes a token from each input place and deposits one in each of its output places.

A PN with n places and m transitions can be represented by the 5-tuple

$$PN = \{P, T, I, O, M_0\}$$

where

- P = {P1, P2, ..., Pn} is the set of places;
- T = {t1, t2, ..., tm} is the set of transitions;
- I is the mapping of $P \times T \rightarrow \mathbb{Z}^+$ such that if there exists *k* input arcs connecting Pi to tj then I(Pi, tj) = *k*;
- O is the mapping of $T \times P \rightarrow \mathbb{Z}^+$ where if there exists *k* output arcs connecting tj to Pi then O(tj, Pi) = *k*; and
- M_0 is the *initial marking* of the PN, that is the initial distribution of tokens in the PN.

Consider the simple example of the PN shown in Figure 1. In this PN:

P = {P1, P2, P3, P4, P5},

T = {t1, t2, t3, t4},

I(P1, t1) = 1   I(P3, t2) = 1   I(P2, t3) = 1   I(P4, t3) = 1   I(P5, t4) = 1

O(t4, P1) = 1   O(t1, P2) = 1   O(t1, P3) = 1   O(t2, P4) = 1   O(t4, P4) = 1
O(t3, P5) = 1

$M_0(P1) = 1$ $M_0(P2) = 0$ $M_0(P3) = 0$ $M_0(P4) = 0$ $M_0(P5) = 0$
where only the non-zero values of the input and output mappings have been given.

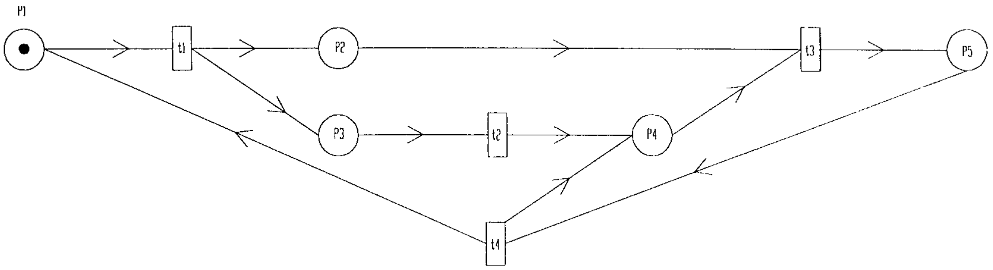


Figure 1: Example Petri net

Initially, transition t1 is enabled and, after it fires, the marking becomes:

$M_1(P1) = 0 \quad M_1(P2) = 1 \quad M_1(P3) = 1 \quad M_1(P4) = 0 \quad M_1(P5) = 0$

Thus the firing of t1 removes a token from P1, and places a token one in both P2 and P3. Now transition t2 is enabled to fire. Marking M_0 is shown in Figure 1.

The order in which transitions of a PN fire is called a *firing sequence.* A PN may have a number of different firing sequences for a given marking, this occurs if more than one transition is enabled during the firing sequence. Consider the PN shown in Figure 2, with the initial marking

$M_0(P1) = 1, M_0(P2) = 0, M_0(P3) = 0.$

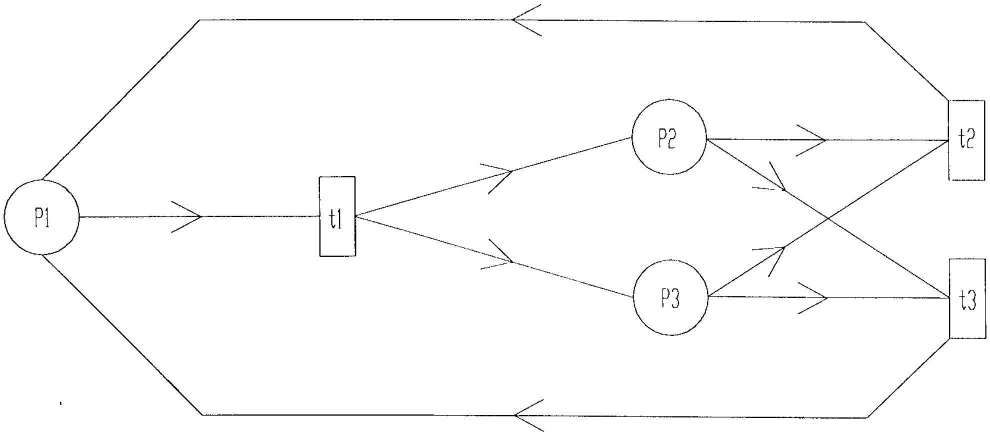


Figure 2: Example Petri net

Some possible firing sequences for this PN are:

- {t1, t2, t1, t2, t1, t2, t1, t2, ...}
- {t1, t3, t1, t3, t1, t3, t1, t3, ...}
- {t1, t2, t1, t3, t1, t2, t1, t3, ...}

There are infinitely many firing sequences for this PN with the initial marking defined above.

2.2 Matrix representation of a Petri net

The structure of a PN with n places and m transitions can be represented by an n × m matrix C, called its *incidence matrix.* The rows in the incidence matrix correspond to places and the columns to transitions, where

$$C_{ij} = O(tj, Pi) - I(Pi, tj).$$

The incidence matrix for the PN in Figure 1 is,

$$C = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

The PN marking can be represented by a vector of size n called the *marking vector,* were the i^{th} element of the vector M is given by M(Pi). Hence if there are *l* tokens in the i^{th} place then the i^{th} element of M will take the value *l*. This means that the marking vector for the initial marking shown in Figure 1 is,

$$M_0 = [1\ 0\ 0\ 0\ 0]^t.$$

If it is known which transition will fire, the resulting marking can be calculated using,

$$M = M_0 + C\,F$$

where F is the *firing vector,* of length m, such that $F_j = 1$ if the j^{th} transition fires, otherwise it is zero. In the example shown in Figure 1, transition 1 is enabled by the initial marking given above, so:

$$M = M_0 + CF = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

This marking vector represents one token in place P2 and one in place P3. In fact any given firing sequence can be represented by the firing vector F, where F_i is the number of times the transition i fires in the sequence. The marking which results from this sequence of firings can now be determined from the initial marking, incidence matrix, and firing vector. For example, in the PN in Figure 1, with the initial marking given above, we can have the firing sequence {t1, t2, t3, t4, t1} which is represented by the firing vector

$$F = [2\ 1\ 1\ 1]^t.$$

The resulting marking can be calculated as follows

$$M = M_0 + CF = \begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix} + \begin{bmatrix}-1 & 0 & 0 & 1\\1 & 0 & -1 & 0\\1 & -1 & 0 & 0\\0 & 1 & -1 & 1\\0 & 0 & 1 & -1\end{bmatrix}\begin{bmatrix}2\\1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\1\\1\\1\\0\end{bmatrix}$$

2.3 Some Petri net properties

There are many different properties defined for PN. This section will briefly outline a few. Other properties can be found in references [12], [27], [32-34], [36], and [38].

(a) *Conflict*: A conflict is said to occur between transitions for a given marking, if more than one transition is enabled at the same time and the firing of any one of these transitions will disable the remaining enabled transitions. Figure 2 shows a PN in which a conflict occurs between transitions t2 and t3 when the marking

$$M = [0\ 1\ 1]^t$$

occurs. When a conflict occurs in a PN, the transition which fires is determined by the firing rules discussed in section 2.5. This is an important property, as it indicates how the sharing of resources effects the system performance. In some literature conflict is referred to as confusion.

(c) *Deadlock*: When deadlock occurs in a PN none of the transitions can fire, halting the execution of the PN. An example of deadlock is the PN in Figure 2 with the marking

$$M = [0\ 1\ 0]^t.$$

A PN which is deadlock free for a given initial marking is known as *live* for that marking. For example the PN in Figure 2 is live for the marking

$$M = [1\ 0\ 0]^t$$

(d) *Reachability*: A marking M_{i+1} is said to be *immediately reachable* from a marking M_i, if there exists a transition enabled by M_i, which on firing will give the marking M_{i+1}. For example, in the PN shown in Figure 2 the marking

$$M_1 = [0\ 1\ 1]^t$$

is immediately reachable from the initial marking,

$$M_0 = [1\ 0\ 0]^t.$$

The marking M_{i+n} is said to be *reachable* from M_i if there exists a firing sequence {ti, tj, tk, ...} such that after the firing of all these transitions, the resulting marking is M_{i+n}.

(e) *Boundedness*: A place of a PN is called *l-bounded* if the number of tokens in the place never exceeds *l*. For the PN in Figure 1, with the initial marking shown in the illustration, places P1, P2, P3, and P5 are one-bounded, however P4 is unbounded. A PN in which all the places are bounded is called a *bounded PN*. The PN in Figure 2 is a bounded PN. The boundedness of places in a PN indicates the maximum number of tokens which can appear in a place. This may correspond to the maximum length of a queue. Hence unbounded places are potential bottle necks. In the special case of the PN in Figure 2, all the places are one-bounded. Such a PN is called *safe*. This is an important property in the modelling of computer hardware, as it indicates that the state of each place can be represented by a one or a zero.

2.4 Timed transitions

In section 2.1 it was stated that at an enabled transition can fire and thus change the marking of the PN in accordance to the input and output arcs. The transitions described earlier needed only to be enabled to fire. This is no longer the case for *timed transitions*. Each transition now takes a stochastically determined period of time, τ say, before it can change the PN marking. The negative exponential distribution is usually used to determine the life time of transition j, where each transition can have a different parameter, τ_j. This is done for simplicity, as it allows the PN to be converted to a CTMC, see section 2.7, and analysed directly. However, any probability distribution can be used to define the time taken for a transition to fire. Note, in this discussion it will be assumed that a negative exponential distribution is used to determine the firing times, but the ideas presented can be extended in most cases to include any type of probability distribution. In a modelling context, timed transitions represent the time taken by the system to perform a given task. Recalling what transitions physically represent, it is also convenient to have transitions which fire in zero time. These are called *immediate transitions* and fire the instant they are enabled. These changes to the PN definition require an expansion of the PN representation to the 6-tuple

$$PN = \{P, T, I, O, M_0, \Omega\}$$

where P, T, I, O, M_0 are as described in section 2.1 and Ω is one of two things: either an *average firing rate* of the transition if it is a timed transition, or a *weight* if it is an immediate transition. Thus Ω has two purposes: it is used in the calculation of the time τ and it determines which immediate transition will fire, if more than one immediate transition is enabled. Note that for some distributions Ω will be defined differently to allow for more parameters.

There are two types of timed transitions. The type used in this document, which requires the transition to be enabled for a period of time before it fires. On firing the transition changes the PN marking as set out in section 2.1. In some literature this type of time delay is defined as an enabling time (see [29]). In the other method the tokens are removed from the input places immediately a transition is enabled and the output tokens are not placed in the output places until the specified period of time has passed (see [29]). An alternative way of representing this second method is to use timed places, whereby tokens are held in places and cannot be used to enable a transition until they have been in the place for a given period of time. This document will consider only the first method defined above.

2.5 Firing rules for resolving conflict

If conflict occurs then a number of strategies can be applied, see [13]. For the situations considered in this document the following strategy will be used:

(a) If all the enabled transitions are timed transitions, then a τ value is determined for each enabled transition and the transition with the shortest time fires.

(b) If all the enabled transitions are immediate transitions, then the Ω values, of each of these transitions, called their weights, are used to determine which one will fire. This is determined in the following manner:

- Let X be the sum of the weights of all the enabled immediate transitions.
- Transition t will fire with a probability of $\Omega(t)/X$.

Thus the transition that fires is determined probabilistically using the weights.

(c) If a combination of immediate and timed transitions are enabled, then only the immediate transitions are considered and they are dealt with as set out above in (b).

It should be noted that a marking in which only timed transitions are enabled is called a *tangible marking*, and a marking with immediate transitions enabled is called a *vanishing marking*. The distinction between these two types of markings is important in the conversion of a PN into a CTMC.

2.6 Generating a reachability tree

A *reachability tree* describes the possible markings of a PN. The root of the tree is the initial marking. Below this marking, each of the possible immediately reachable markings are listed. Directed arcs going from the initial marking to each of the immediately reachable markings are drawn and labelled with the transition required to reach the specified marking. This process is then repeated for each of the markings generated. If the marking to be generated is equivalent to one which appears earlier in the tree, then the generating marking is connected to the earlier marking by an arc labelled with the appropriate transition.

Consider the PN in Figure 3 with the initial marking

$$M_0 = [1\ 0\ 0\ 0\ 0]^t.$$

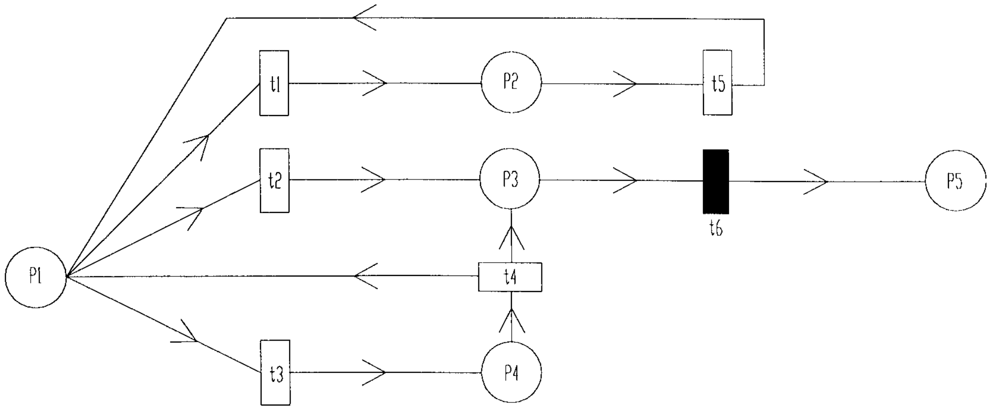


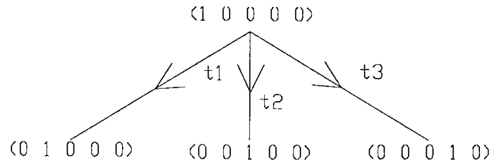
Figure 3: Example Petri net

In this PN, the transition t6 is considered to be an immediate transition, whereas the other four are timed transitions. When conflict occurs between immediate and timed transitions, the firing rules set out in section 2.5 must be observed. That is, the immediate transitions will always fire before the timed ones, so the markings which correspond to the timed transitions are not reachable and so do not appear in the reachability tree. The generation of the reachability graph for the PN in Figure 3 is as follows:

(a) Step 1: Transitions t1, t2 and t3 are the only enabled transitions for initial marking, so the immediately reachable markings are

$$M_1 = [0\ 1\ 0\ 0\ 0]^t, M_2 = [0\ 0\ 1\ 0\ 0]^t \text{ and } M_3 = [0\ 0\ 0\ 1\ 0]^t$$

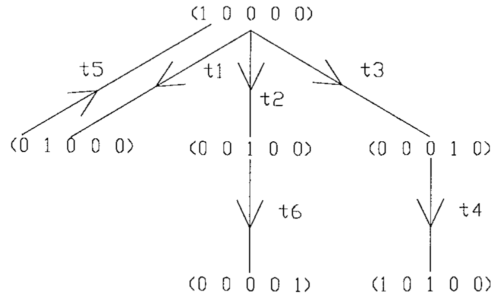
thus, these three markings are listed as the children of the root. The appropriate label is then placed on each of the arcs connecting the initial markings to the new marking. The tree now takes the form:



(b) Step 2: Now each of the immediately reachable markings from M_1, M_2 and M_3 are listed in the tree, these are

$$M_4 = [1\ 0\ 0\ 0\ 0]^t, M_5 = [0\ 0\ 0\ 0\ 1] \text{ and } M_6 = [1\ 0\ 1\ 0\ 0]^t$$

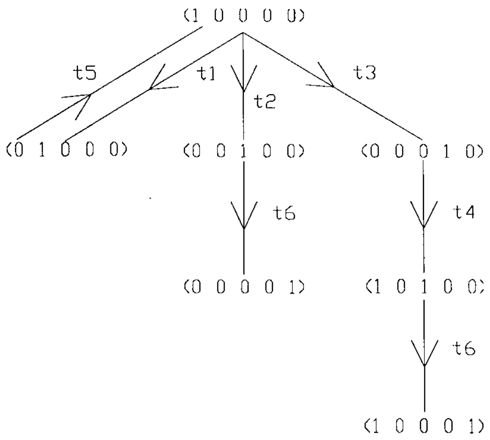
respectively. Since the only marking reachable from M_1 is M_4 which is equivalent to M_0, no marking is listed below M_1. Instead another arc is drawn between M_0 and M_1, this time going from M_1 to M_0 and labelled t5. However, markings M_5 and M_6 are added to the tree on the next level. The reachability tree becomes:



(c) Step 3: When the PN has the marking M_5 it is in deadlock, and so no more markings can be reached. Thus, this branch of the tree has reached its end. However, the marking

$$M_7 = [1\ 0\ 0\ 0\ 1]^t$$

is reachable from M_6, so this branch continues. It should be noted that transitions t1, t2 and t3 are enabled when the PN has the marking M_5, but never fire, as they are timed transitions, whereas t5, which is also enabled, is an immediate transition. Thus, the markings produced by the firing of the time transitions t1, t2 or t3 do not occur. The tree is now:



This process continues on and Figure 4 shows the resulting tree after eight steps. It should be noted that, for the PN specified, this process can continue indefinitely. This necessitates the definition of a new type of tree, the *coverability tree.* In a coverability tree, any set of markings which differ only by the number of tokens found in unbounded places, are represented by one marking. A w is placed in the unbounded place/s indicating the number of tokens in that place to be unbounded. The w corresponds to any element of the set $\mathbb{Z}^+$. This is best illustrated by an example. Figure 5 shows the coverability tree for the PN shown in Figure 3. The coverability tree is designed in a similar way as the reachability tree, for more details see [34].

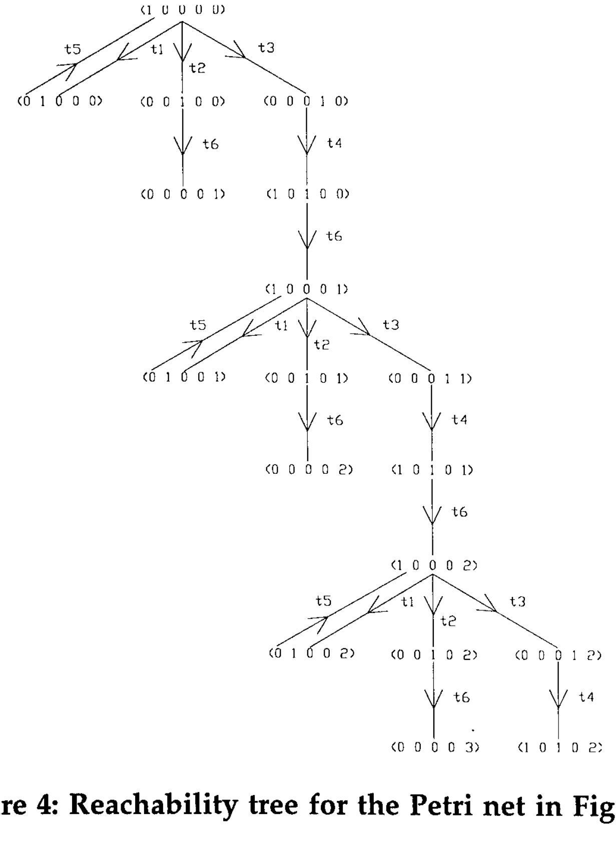


Figure 4: Reachability tree for the Petri net in Figure 3

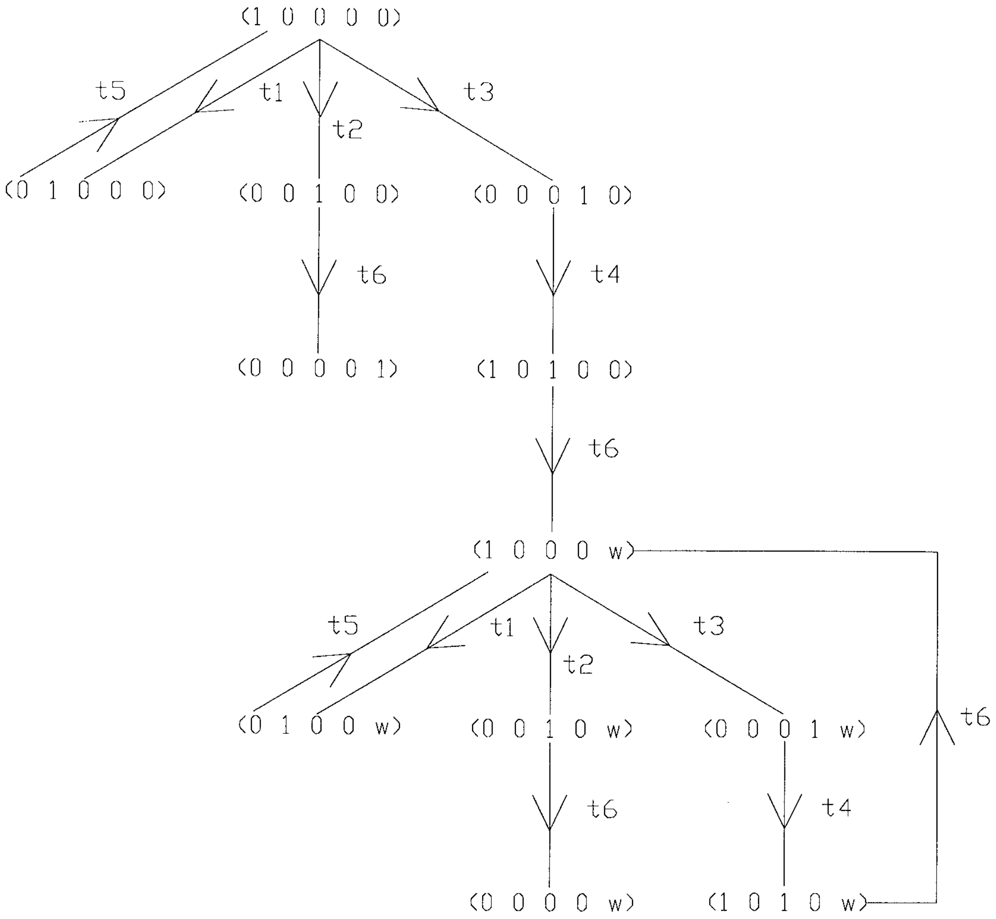


Figure 5: Coverability tree for the Petri net in Figure 3

By constructing a reachability or coverability tree, the set of reachable markings is easily obtained. A reachability tree can be used to determine safeness, boundedness, conservation, and reachability of a PN. The process in which this can be done is outlined in [34].

2.7 The use of the Petri net matrix representation in analysis

Linear algebra techniques can be applied to the matrix representation of a PN to solve the problems of conservation, reachability, coverability, boundedness, and deadlock. For example, consider the determination of whether or not a given marking is reachable from a given initial marking, for a defined PN. That is, C (incidence matrix), M_0 (initial marking) and M (desired marking) are given, and F (the firing vector) is unknown. This problem can be solved using the following theorem, adapted from elementary linear algebra, see [30].

Consider the system of equations

$$C\,F = M - M_0 = \Delta M.$$

Then only one of the following must hold:

(a) If the rank of the augmented matrix $[C\ |\Delta M]$ is greater than that of C, then there is no solution to the system of equations.

(b) If the rank of [C |ΔM] is equal to the rank of C and the rank of C equals the number of unknowns, then there is a unique solution to the system of equations.

(c) If the rank of [C |ΔM] is equal to the rank of C and the rank of C is less than number of unknowns, then there exists an infinite number of solutions to the system of equations.

For the PN shown in Figure 1, with M_0 and C given in section 2.2 it can be determined whether or not the markings

$$M_1 = [0\ 1\ 1\ 1\ 0]^t \text{ and } M_2 = [0\ 0\ 0\ 0\ 0]^t$$

are reachable. The rank of C is 4. Taking M_1 first we get,

$$\Delta M_1 = M_1 - M_0 = [-1\ 1\ 1\ 1\ 0]^t$$

$$\therefore\ [C|\Delta M_1] = \left[\begin{array}{cccc|c} -1 & 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

which also has a rank of 4. Thus the marking M_1 is reachable from the initial marking M_0 via a unique firing vector F. This firing vector is

$$F = [2\ 1\ 1\ 1]^t.$$

It should be noted that, although in this case F represents the firing sequence, {t1, t2, t3, t4, t1}, it is not always possible to construct such a sequence from a derived firing vector, making it impossible to reach the desired marking although algebraically it appears to be possible. This is explained more thoroughly later in this section. Consider now M_2 where

$$\Delta M_2 = M_2 - M_0 = [-1\ 0\ 0\ 0\ 0]^t$$

$$[C|\Delta M_2] = \left[\begin{array}{cccc|c} -1 & 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right].$$

This augmented matrix has a rank of 5, which is larger than the rank of C, thus the marking M_2 is not reachable from M_0.

It should be noted that although matrix analysis gives the firing vector required to reach the desired marking, the firing sequence represented by the vector may be impossible. Consider the marking

$$M = [1\ 0\ 1\ 0\ 0]^t.$$

The above analysis gives the firing vector

$$F = [1\ 0\ 1\ 1]^t$$

which corresponds to a firing sequence involving transitions t1, t3, and t4. However, t3 is only enabled when there is a token in both places P2 and P4. This means t2 must fire, but t2 is not described as firing by F. Thus, there exists no possible firing sequence corresponding to F. This downfall in matrix analysis results from the fact that firing vectors do not represent in any way the order in which transitions fire. Thus with this type of analysis, it is important to make sure the results make sense not only mathematically, but also within the PN definition.

2.8 Converting a Petri net into a continuous time Markov chain

In [14] there is a short algorithm which allows a PN to be converted into a CTMC. Defining NM as the set of new markings, RS as the reachability set, E(m) as the set of transitions which can fire when the PN has the marking m (it should be noted that if an immediate transition is enabled, then any enabled timed transitions are considered not to be enabled), Q is the probability of moving between states and P the initial state of the CTMC. The algorithm for converting the PN defined by S into the CTMC defined by RS, Q and P(0) is:

```
•   input : $S = (P, T, I, O, M_0, \Omega)$
•   $NM := \{M_0\}$, $RS := \{M_0\}$
•   While $NM \neq \{\varnothing\}$ do
    •   let $m \in NM$
    •   $NM := NM - \{m\}$
    •   For all $t \in E(m)$ do
        •   let m′ be the marking obtained after the firing of t in m
        •   store $Q(m, m', \Omega(t, m))$
        •   if $m' \notin RS$ Then   $NM = NM \cup \{m'\}$
                                     $RS = RS \cup \{m'\}$
•   $P(0) = (1\ 0\ 0\ \ldots\ 0)$
```

This algorithm constructs the sets RS, Q, and P(0). RS contains the states of the CTMC, Q the probability of moving from one state to the next, and P(0) the initial probability vector of the chain. It should be noted that only PN with a finite number of reachable markings can be converted into a CTMC with this algorithm. The generator matrix is constructed in such a way that the vanishing markings are listed first, and then the tangible markings. From here the CTMC can be analysed to get information about the PN. For more information about this type of analysis see [14].

2.9 Coloured Petri nets

In many modelling situations there is a need to distinguish between different elements in the system. For example there may be a need to distinguish between different types of resources, customers, information, etc. This is easily achieved by being able to differentiate between tokens, that is, introduce *coloured tokens* into the PN model. Each coloured token represents a different physical identity in the system being modelled. This allows for varying *firing modes* of transitions, each mode depending on the colours of the tokens present in the input places. These firing modes can also have different output places and different output token colours. A PN with these properties is called a *coloured Petri net* (CPN) and is defined by the 8-tuple

$$\{C, P, T, K, \Phi, I, O, M_0, \Omega\}$$

where

- C is the set of coloured tokens. Each token may be a complex data structure which reflects the type of infromation stored at the places where the token can be located;
- P is the set of places;
- T is the set of transitions;
- K maps each place into the set of possible token colours that can be found in the place. Thus $\forall\ p \in p$, $K(p) \subseteq C$ defines the possible token colours of place p;
- Φ maps each transition into the set of possible firing modes. That is, $\forall\ t \in T\ \Phi(t)$ contains the possible firing modes of transition t;
- $I(p,t)_{c,\phi}$ is a mapping of $c \times \phi \rightarrow \mathbb{Z}^+$, where $c \in K(p)$ and $\phi \in \Phi(t)$, which defines the input arc inscriptions;
- $O(t,p)_{\phi,c}$ is a mapping of $\phi \times c \rightarrow \mathbb{Z}^+$, where $c \in K(p)$ and $\phi \in \Phi(t)$, which defines the output arc inscriptions;
- M_0 is the mapping of $K(p) \rightarrow \mathbb{Z}^+$, which describes the initial distribution of coloured tokens in the CPN. Thus, if initially l tokens of colour i are present in place p, then $K(p)_i = l$; and
- Ω defines either an average firing rate of the transitions firing mode if it is a timed transition, or a weight if it is an immediate transition.

The CPN definition given above is an extension of the CPN definition given in [31] and [18-19]. Consider now the CPN in Figure 6.

In this CPN:

$C = \{a, b, c, d, e\}$

$P = \{P1, P2, P3, P4, P5, P6\}$

$T = \{t1, t2, t3, t4\}$

$K(P1) = K(P3) = \{a\}$, $K(P2) = K(P4) = \{c, b\}$, $K(P5) = \{d\}$, $K(P6) = \{e\}$

$\Phi(t1) = \{1\}$, $\Phi(t2) = \{2, 3\}$, $\Phi(t3) = \{4\}$, $\Phi(t4) = \{5\}$

$I(P1,t1)_{a1} = 1$, $I(P2,t2)_{b2} = 1$, $I(P2,t2)_{c3} = 1$, $I(P3,t3)_{a4} = 1$,$I(P4,t3)_{b4} = 1$, $I(P4,t4)_{c5} = 1$, $I(P5,t5)_{d5} = 1$

$O(t1,P3)_{a1} = 1$, $O(t2,P4)_{b2} = 1$, $O(t2,P4)_{c3} = 1$, $O(t3,P5)_{d4} = 1$, $O(t4,P6)_{e5} = 1$

$M_0(P1)_a = 1$, $M_0(P2)_b = 0$, $M_0(P2)_c = 1$, $M_0(P3)_a = 0$, $M_0(P4)_b = 1$, $M_0(P4)_b = 0$, $M_0(P5)_d = 0$, $M_0(P6)_e = 0$

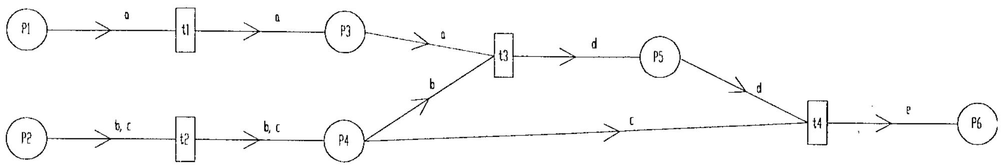


Figure 6: Example of a coloured Petri net

CPN have similar firing rules to PN only now the colour of the tokens in the input places must be considered. Thus firing mode $F(tj)_k$ is enabled when each of the input places of transition tj have the correct colours, ie for all $Pi \in P$ and $h \in C(Pi)$ $I(Pi,tj)_h \leq M(Pi)_h$. If this is true then the transition may fire, removing the specified tokens from each input place, and placing the relevant coloured tokens in the output places.

As with PN, CPN can be represented by matrix notation. The CPN with n places and m transitions can be represented by the n × m block matrix, C, again referred to as the incidence matrix. Note that the above notation for O and I can easily be converted into matrix form, where for example, O(tj,Pi) is a matrix of size $|K(Pi)| \times |\Phi(tj)|$, in which the rows represent the token colours and the columns the firing modes. Thus the block matrix representing the change to the number of coloured tokens in Pi when fires is defined by

$$C(Pi,tj) = O(tj,Pi) - I(Pj,ti).$$

The marking of the CPN can be represented by the n × 1 block vector where M(Pi) is a $|K(Pi)| \times 1$ vector, in which each element relates to the number of the given coloured tokens which can appear in place Pi. The firing sequence of a CPN can be represented by an m × 1 block vector, in which each element is a $|\Phi(tj)| \times 1$ vector, which corresponds to the number of times a given firing mode of transition tj fires, in the firing sequence. As with PN, if we are given the CPN incidence matrix, initial marking vector, and the firing vector, the resulting marking can be calculated from,

$$M = M_0 + C\,F$$

Consider the CPN in Figure 6. For this example

$$C = \begin{bmatrix} -C1 & 0 & 0 & 0 \\ 0 & -C2 & 0 & 0 \\ C1 & 0 & -C3 & 0 \\ 0 & C2 & -C4 & -C5 \\ 0 & 0 & C3 & -C5 \\ 0 & 0 & 0 & C5 \end{bmatrix}$$

where

$$C1 = C3 = C5 = [1],\ C2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } C4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

and

$$M_0 = [M1\ M2\ M3\ M4\ M5\ M6]^t$$

where

$$M1 = [1],\ M2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ M3 = M5 = M6 = [0],\ \text{and}\ M4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

Given that firing mode 1 of transition t1 fires, firing mode 3 of transition t2 fires and firing mode 4 of transition t3 fires, the firing vector is

$$F = \begin{bmatrix} F1 \\ F2 \\ F3 \\ F4 \end{bmatrix}$$

where

$$F1 = F3 = [1],\ F2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \text{and}\ F4 = [0].$$

So the resulting marking can be calculated from

$$M = M_0 + C\ F$$

which gives the marking

$$M = \begin{bmatrix} M'1 \\ M'2 \\ M'3 \\ M'4 \\ M'5 \\ M'6 \end{bmatrix}$$

where

$$M'1 = M'3 = M'6 = [0],\ M2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ M'4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \text{and}\ M'5 = [1].$$

It should be noted that the extensions presented in this document are only a sample of the many extensions which can be made to PN. More may be found in literature on PN, such as [6], [12] and [44]. Most PN extensions can be modelled by ordinary PN. For example CPN can be converted into PN, as shown later when the CPN in section 4, which is converted to a PN in section 5.1.

3 A COMMAND AND CONTROL SYSTEM

Consider the air defence of a significant asset. Such an asset may be an important runway or a large storage facility. To protect the asset from air attacks the position is equipped with two operator guided surface-to-air missile (SAM) kits, a radar, a command centre, and communication links between the command centre and each SAM operator.

3.1 System layout

Consider the case when the air threat comes from a 180° arc, stretching from north to south in a clockwise direction and the terrain surrounding the asset does not impede either the range of the SAM or the radar. The SAM range is less than that of the radar, so the full detection range of the radar cannot be covered. The SAM sites are positioned to give maximum coverage of the area to protect. These positions are shown as SAM 1 and SAM 2 in Figure 7. The SAM range, detection line and line of weapon release are also indicated. All enemy aircraft must be destroyed before they reach the line of weapon release or they will destroy the asset. The area surrounding the asset has been divided into three sectors. Each sector has a unique set of SAM sites which can effectively fire at aircraft detected in it. Note that the sectors are arranged so that they coincide with the intercept between the line of weapon release and the SAM range. Table 1 shows which of the different SAM sites are effective in each of the sectors.

3.2 Sequence of events

Each time an aircraft is detected by the radar the following events occur:

(a) The command centre determines which SAM site it will be assigned to. This is accomplished by taking into account the sector the aircraft is in and the current availability of SAM sites. SAM sites which are assigned aircraft are considered unavailable. If none of the available SAM sites can effectively deal with the aircraft, then the aircraft is placed in a queue. Such an aircraft is assigned when an appropriate SAM site becomes available.

(b) Once the command centre has determined which SAM sites will be assigned aircraft, the aircraft flight path is passed to the chosen SAM site. This is done through the communication link between each SAM site and the command centre. The aircraft now becomes the sole responsibility of the SAM operator.

(c) The SAM operator locates the aircraft and aims his weapon at it.

(d) Once the aircraft has been acquired by the SAM operator, an assessment is carried out to determine if it is friend or foe.

(e) If the SAM operator decides that the aircraft is not a threat to the asset, then it is allowed to pass. The SAM site returns to its ready status, and the command centre is informed of the SAM site's availability.

(f) However, if the aircraft proves to be a threat, the SAM operator tracks the aircraft and fires at it. The operator then guides the missile to the aircraft destroying it. Once the enemy aircraft has been destroyed, the SAM operator reloads and returns to the ready status. He then communicates his availability.

This sequence of events is followed for each detected aircraft.

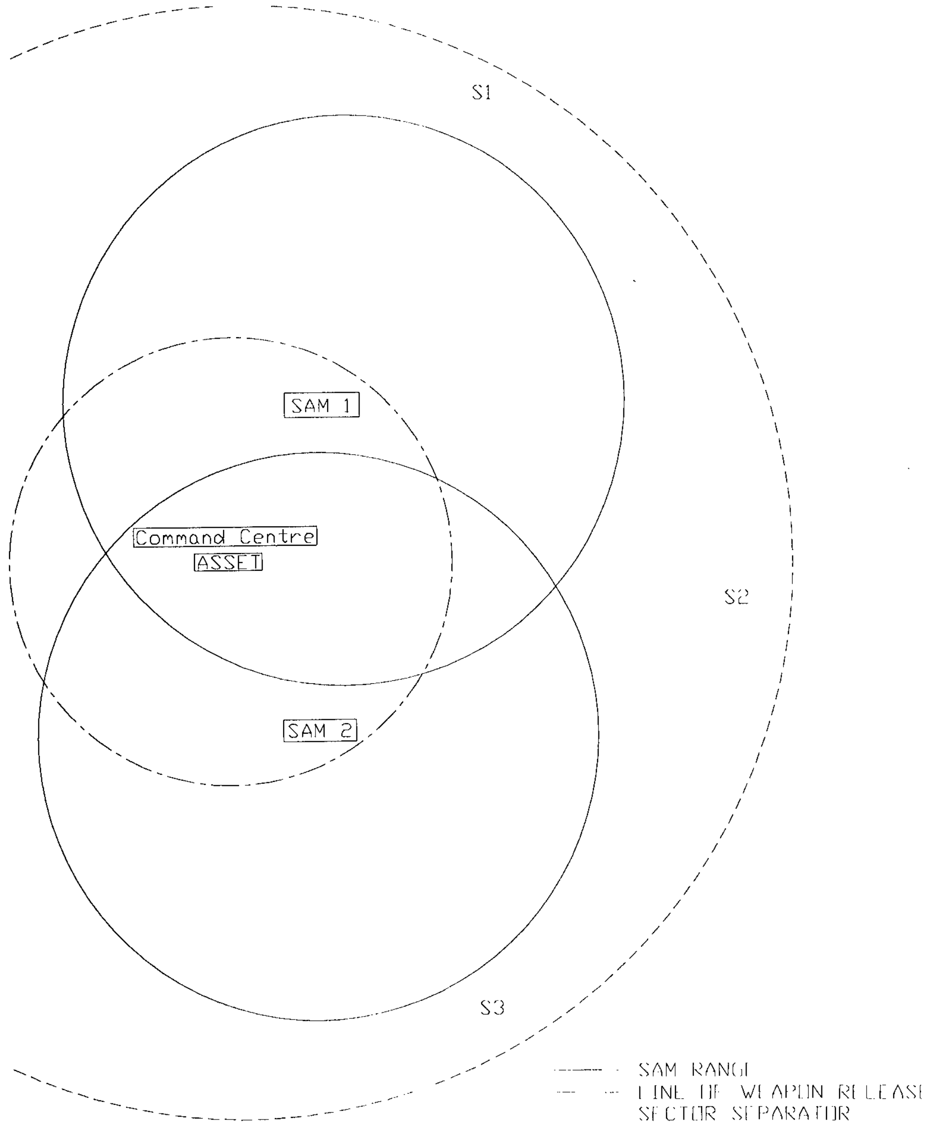


Figure 7: Defensive layout for air defence of an asset

Table 1: Sets of SAM sites which are effective in given sectors.

Sector Number	Effective SAM sites
S1	SAM 1
S2	SAM 1, SAM 2
S3	SAM 2

4 THE CPN MODEL

This section will outline how a CPN can be used to model the system described in section 3. To avoid enlarging the model, which would make it harder to understand and relate to other

C2 systems, some assumptions will be made. In section 5 this model will be used to analyse time delays in the system and look at a method of determining cost effective changes that can be made.

4.1 Assumptions

As this model is intended only as a means of demonstrating the use of CPN's as a modelling tool for C2 systems, a number of assumptions have been made to reduce the size of the model. These are:

(a) All the aircraft stay in the same sector once the radar has detected them. Thus the aircraft can be covered by the same SAM site throughout its flight, removing the need to transfer the aircraft to a different SAM site if they move into a sector where the selected SAM site is ineffective.

(b) All aircraft are detected by the radar when they reach the detection range. This means the SAM operators do not have to search for aircraft that may have evaded the radar. Thus SAM operators only search for aircraft to which they have been assigned by the command centre.

(c) The probability of a kill, once a SAM site fires, is 1. Therefore if an aircraft is fired upon before passing the line of weapon release, it will be destroyed before it can deliver its ordinance and destroy the asset.

(d) All the aircraft fly at approximately the same altitude. This makes the SAM range topographically equivalent for each aircraft, fixing the effective range (see Figure 7) for each aircraft.

(e) All aircraft designated as friendly are not a threat to the asset and only enemy aircraft as shot down. This model's purpose is to study the time delays involved in the system, and is not concerned with friendly fire or deception techniques.

(f) The perceived threat of the aircraft is independent of the sector that the aircraft is detected in.

Without these assumptions considerably more detail would be added to the model, making it even more complicated, and harder for the reader to relate the concepts demonstrated to other C2 systems. For example, assumption 5 can be removed by dividing the sky around the asset into more sectors. This allows for aircraft of different altitudes to be present in the model, as these will correspond to different sectors. As before, each sector would contain a unique set of effective SAM sites. There would be more sectors that needed to be considered, some of which would be height dependent.

4.2 Model Description

The CPN of this system is shown in Figure 8. It involves 11 places, 17 transitions, and 5 different coloured tokens. The five tokens are S1, S2, S3, M1 and M2. S1, S2, and S3 correspond to an aircraft arriving in sector 1, 2, or 3, respectively. M1 and M2 represent the current process which SAM 1 and SAM 2, respectively, are performing. To explain this CPN, a brief outline of what each transition represents in the problem system and how they effect the distribution of the tokens among the places, is given.

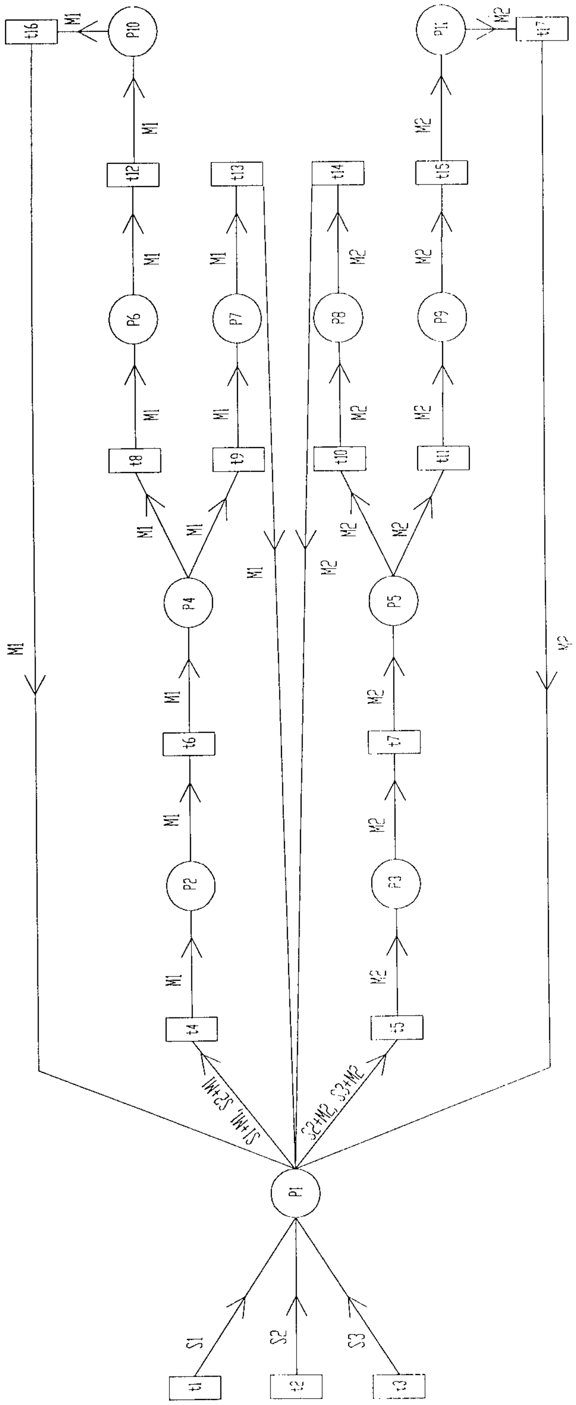


Figure 8: A coloured Petri net model of the air defence of an asset

4.2.1 The arrival of the aircraft: Transitions t1-t3

Transitions t1-t3 correspond to the detection of aircraft. Each transition corresponds to the presence of an aircraft in a different sector. For example an S2 token in place P1 corresponds to a threatening aircraft in section S2.

4.2.2 Aircraft assignment: Transitions t4-t5

Transitions t4-t5 relate to the assigning of an aircraft to a selected SAM site, and the passing of the flight path information. Aircraft can only be assigned to a SAM site if there is an available site which can destroy it before it reaches the line of weapon release. For example, if an S1 token is in place P1 then the aircraft can only be assigned to SAM 1. Hence t4 will only fire if an M1 token is also present in place P1. The firing of this transition removes the M1 and S1 tokens from P1 to and places an M1 token in place P2.

4.2.3 Location and investigation of aircraft: Transitions t6-t7

Transitions t6-t7 correspond to the time spent by the SAM site operator to find the aircraft in the sky, train the weapon system on the target, investigate it, and decide its status.

4.2.4 SAM operator assessment: Transitions t8-t11

Transitions t8-t11 are immediate transitions relating to the resulting decision made by the SAM operator about the incoming aircraft. The firing of t8 or t11 means that the aircraft is a threat to the asset, and t9 or t10 relate to the aircraft being allowed to pass.

4.2.5 Missile fire: Transitions t12 and t15

Transitions t12 and t15 correspond to the SAM operator firing the missile and tracking the aircraft to guide the missile onto it.

4.2.6 Return to ready status: Transitions t13-t14 and t16-t17

Transitions t13-t14 relate to the SAM site allowing the aircraft to pass, the SAM site being returned to the ready status, and the communication of its readiness to the command centre. Transition t16-t17 correspond to the SAM site reloading, returning to the ready status, and communicating the site's availability.

Transitions t12 and t16, and t15 and t17 have been separated, as the analysis being carried out is concerned with the time taken to destroy the aircraft. Therefore, there needs to be a place which represents the state in which the aircraft has just been destroyed. For SAM 1 this state is represented in the model by a token in place P10, and for SAM 2, a token in place P11. So, although the actions which relate to t12 and t16 (t15 and t17) may logically be grouped together, for analysis reasons they must be separated.

The immediate transitions t8-t11 are used as generators of information. The weights placed on these transitions relate to the probability of the aircraft being a friend or a foe. The timed transitions correspond to the time it takes for an event to occur or a task to be performed.

5 MODEL ANALYSIS

The CPN model described in section 4 will now be analysed to study the efficiency of the air defence system it represents. There are a number of packages such as Great SPN, [7], and SPNP, [8], available, which do PN analysis. These packages concentrate on equilibrium probabilities. They also work on the general principle of converting the PN into a finite CTMC, and analysing this chain. For the CPN described above, this is not possible, as place P1 is unbounded and thus converts to a Markov chain with an infinite number states. This can be prevented by using inhibitor arcs with a given multiplicity from P1 to t1, t2, and t3, or by initially starting with only k tokens in a newly defined input place to transitions t1, t2, and t3. If these restrictions are made then the above packages could be used to obtain long term equilibrium information. Unfortunately for the analysis being carried out in this document only transient properties are of interest. These properties are unattainable from packages like SPNP and Great SPN. For this reason the CPN model was simulated.

There are two possible approaches which can be taken in simulating a CPN, either direct simulation of the CPN, or conversion of the CPN into a PN and simulation of the PN. The PN model of this C2 system is shown in Figure 9. This removes the need to have block matrices as described in section 2.9. The simulation is used to determine which of five possible improvements to the system is most cost effective.

5.1 The CPN to PN conversion

In converting the CPN given in section 4 to a PN a process of unfolding is carried out. The representation of P1, t4, and t5 in Figure 8 were changed as follows:

(1) Place P1 is now represented by five places: P1a, P1b, P1c, P1d, and P1e. A token in place P1a, P1b, or P1c corresponds to an aircraft in sector S1, S2, or S3, respectively. For example, a token in P1a means there is an aircraft in sector S1, and this corresponds to a S1 token in place P1 of the CPN. A token in place P1d or P1e of the PN corresponds to the availability of SAM sites one and two respectively. Thus a token in P1d or P1e is the same as the presence of an M1 or M2 token, respectively, in P1 of the CPN in Figure 8.

(2) Transition t4 of the CPN is represented in the PN by two transitions: t4a and t4b. Each of these transitions correspond to one of t4's firing modes. t4a relates to an aircraft in sector one being assigned to SAM 1, and t4b a sector two aircraft.

(3) As with transition t4, t5 of the CPN is represented by two transitions in the PN: t5a and t5b. Once again, these relate to the two firing modes of t5. t5a to an aircraft in sector two being assigned to SAM 2, and t5b an aircraft in sector three.

The PN shown in Figure 9 can be represented by a 6-tuple as set out in section 2.1. It is this representation which is used in simulating the PN.

5.2 The simulation

An event-stepping simulation of the PN in Figure 9 was written in Turbo Pascal. The Erlang-5 probability distribution was used to determine the time to fire of the timed transitions. This distribution was chosen as it has a coefficient of variation considerably less than one and so it seems to model well the duration of these activities. The immediate transitions fire in accordance with the firing rules expressed in section 2.5.

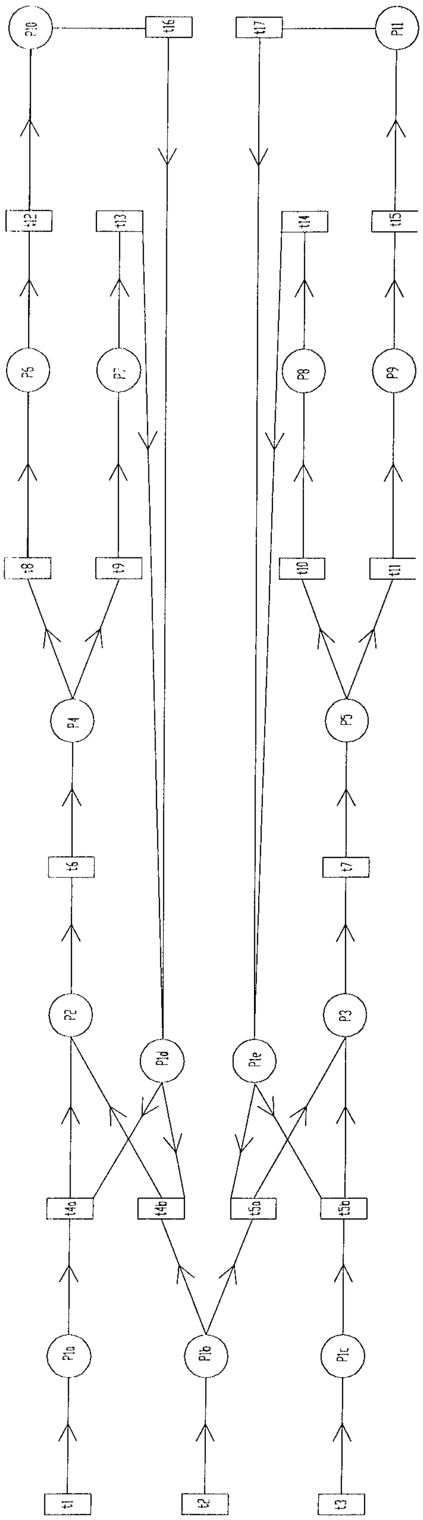


Figure 9: Petri net of the air defence system

The sequence of events which the program follows is:

(1) The program checks which transitions are enabled. As the marking of the PN is represented in a vector, and the input arcs in a matrix, the program simply compares the current marking with the number of tokens required in each place, to enable the transition currently being tested. At the beginning of the simulation there is one token in place P1d and one in place P1e, so only t1, t2, and t3 are enabled.

(2) The program finds which transition is scheduled to fire next. Initially the newly enabled transitions are placed in the event schedule. Then the transitions which were disabled by the last transition fired are removed from the event schedule. Finally, the transition which will fire next is determined using the firing rules outlined in section 2.5.

(3) The final process is the firing of the transition. This updates the marking according to the input and output arc matrices. This process also controls the aircraft. The aircraft are placed in a link list as they arrive and their time of arrival is recorded. As transitions fire, the aircraft's position in the PN is updated, so that if it is destroyed, the time till its destruction can be calculated.

The processes above are repeated, until the time taken to destroy an enemy aircraft is greater than the maximum allowed time; that is the asset has been destroyed. The total simulation run time and the number of aircraft destroyed is then recorded for analysis.

5.3 Trial system improvements

In analysing the model six different configurations of the C2 systems were considered. Initially data was gathered on the C2 system as it is described above. Then five improvements to this initial system were considered to determine which would result in the greatest system efficiency. The changes where:

(i) In Trial 1 an artificial decision maker is placed in the command centre. It decides which SAM site deals with the incoming threat. This decision is almost instantaneous and the only time now involved in transitions t4 and t5 is the communication of the aircraft flight information to the selected SAM site.

(ii) Trial 2 corresponds to the SAM operators being given a better method of locating the detected aircraft, such as improved aircraft flight information. This means that the time taken by the operator to find the aircraft is reduced, thus reducing the average firing time of transitions t6 and t7.

(iii) Trial 3 relates to the use of a missile that is about twice as fast as the original one, reducing the post-firing guiding time, that is, changing the parameters of t12 and t15.

(iv) In Trial 4 fire and forget missiles are used, once again reducing the mean firing time of t12 and t15.

(v) Trial 5 relates to the use of a multi-missile launcher. This does not mean a SAM site can engage more than one aircraft but does remove the need for a SAM site to reload before being assigned a new aircraft. This reduces the average firing times of transitions t16 and t17.

The model presented in section 5.1 was analysed to determine the effect of each of these changes to the C2 system. The transition rates are given in Table 2. Only the changes made to the initial firing rates are shown in the case of the five different trials.

The critical quantity was how long it took a token to travel from place P1a, P1b, or P1c to either place P10 or P11. That is, the time take for a detected enemy aircraft to be destroyed. If this value was greater than 100s, the asset was considered destroyed. The number of aircraft destroyed and current run time when the asset was destroyed were then stored and the simulation run again. This process was continued for the six different configurations given above. Each trial affects the firing times of the transitions' firing modes. It should be noted that the times taken for the described events to occur are arbitrary and in no way relate to any particular existing system. They are used for illustration only.

The results of the five upgrades presented and the initial scenario, are given in the next section.

5.4 Model Results

The results for each of the trials and the initial run are shown in Table 3. It should be noted that each of the sets of data involves 1000 runs. A run always starts with the same initial marking and an empty event queue, and ends when the asset is destroyed. Included in Table 3 is a 90% confidence interval of the mean, calculated from the standard normal distribution.

5.5 Model Conclusion

Table 3 clearly shows the effect on the system efficiency for each of the proposed changes. Consider now, which of the changes suggested gives the best result for the investment made in implementing the change. Table 3 shows that all of the changes suggested above benefit the air defence system described. So the question is, which of the changes is most cost effective. First a ranking of the changes in order of most advantageous to least favourable must be established. Putting aside the question of confidence intervals, this order is:

- Fire and forget missiles, Trial 4.
- Artificial decision maker in the command centre, Trial 1.
- Better location method, Trial 2.
- Faster missiles, Trial 3.
- Multi-missile launcher, Trial 5.

A possible ranking of implementation costs, in order of increasing cost is:

- Better location method, Trial 2.
- Artificial decision maker in the command centre, Trial 1.
- Multi-missile launcher, Trial 5.
- Faster missiles, Trial 3.
- Fire and forget missiles, Trial 4.

Carrying out a t-test with 90% confidence, Trial 1 and Trial 4 give the greatest improvement. Combining this information with the above cost ranking would indicate that the most cost effective change is to introduce the use of an artificial decision maker in the command centre, Trial 1.

Table 2: Transition firing rates for the air defence Petri net model

TRANSITION	FIRING RATE (s^{-1})					
	Initial	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
1	0.005					
2	0.005					
3	0.005					
4a	0.05	0.1				
4b	0.04	0.1				
5a	0.04	0.1				
5b	0.05	0.1				
6	0.05		0.0667			
7	0.05		0.0667			
8	2					
9	1					
10	1					
11	2					
12	0.1			0.2	1	
13	0.05					
14	0.05					
15	0.1			0.2	1	
16	0.0333					0.05
17	0.0333					0.05

Table 3: Simulation results

TRIAL	TIME			COUNT		
	Mean	Confidence Interval for the Mean	Standard Deviation	Mean	Confidence Interval for the Mean	Standard Deviation
Initial	1761	(1682,1840)	1518	17.21	(16.42,18.01)	15.29
1	2961	(2815,3107)	2815	29.82	(28.34,31.30)	28.50
2	2701	(2574,2828)	2442	27.34	(26.02,28.66)	25.33
3	2430	(2311,2549)	2282	24.24	(22.03,25.49)	23.37
4	3124	(2978,3270)	2813	31.59	(30.08,33.10)	28.95
5	2392	(2281,2503)	2143	24.10	(22.95,25.25)	22.15

6 CONCLUSIONS

This document has introduced a CPN notation that can be used to model C2 systems. The advantages of using CPN in the modelling of C2 systems has been shown though the use of an example. This example shows how the CPN of a C2 system can be analysed using computer simulation. It also illustrates how easily a CPN model can be changed to test variations to the system. It should be noted that PN were designed to model discrete event systems with concurrence and resource sharing and so are ideal for the modelling of C2 system. The extension of PN to allow coloured tokens has meant that more complex systems can be modelled without the graphical representation becoming unmanageable. It is the author's opinion that PN are the ideal tool for modelling C2 systems.

One of the main draw backs of using PN is the fact that there is not much work being carried out in the area of transient analysis. Since in many cases C2 systems do not reach an equilibrium state there is no way of directly analysing a C2 PN model to get the information required. This means that simulations must be constructed to generate the results needed. It also means that there does not exist packages that can be used to analysis the transient nature of PN models. This is one of the areas that research is currently being pursued by Systems Simulation and Assessment Group of Information Technology Division and it is hoped that this research can be combined to produce a package capable of transient analysis not only by use of simulation but also through direct methods.

Another area in which PN are being applied by Systems Simulation and Assessment Group is in the modelling of decision processes in C2 systems. In this case PN models will be used to supplement real decision makers, both groups and individuals, in a large distributed interactive C2 simulation.

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Fred D.J. Bowden

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19. ABSTRACT

Petri nets are directed graphs which were designed to model discrete event systems with concurrence and resource sharing. This makes them a useful method of graphically representing command and control systems. In this document Petri nets and some of their extensions are explained in detail. Coloured Petri nets, an extension of Petri nets, with timed transitions are used to model a command and control system.